

## ON THE RELATIONSHIP BETWEEN THE SURFACE STRESS AND THE WIND

MAURICE B. DANARD and STEPHEN CLODMAN

University of Waterloo, Waterloo, Ontario, Canada

## ABSTRACT

The surface stress is computed by the method of geostrophic departures using 23 days of double theodolite wind observations at Shilo, Manitoba. The main results are as follows. Relations between the stress and the geostrophic wind fit these data better than do similar expressions using the surface wind. The stress varies linearly, rather than quadratically, with wind speed. For a given surface or geostrophic wind speed, the stress increases with increasing Richardson number and warm air advection.

## 1. INTRODUCTION

This paper is concerned with computing the stress  $\tau_0$  (force per unit area) exerted by the atmosphere on the earth. Some simple relationships that could readily be incorporated into a numerical weather prediction model are examined.

This study was to some extent motivated by a recent study of one of the authors (Danard 1969). In the latter paper, it was demonstrated that a cyclone would experience significant changes in its velocity fields and rate of frictional filling as it moved over a surface of varying roughness (for example, land to sea). Consequently, if a numerical model is to simulate the behavior of such cyclones, the surface stress must be carefully calculated.

In section 2, an independent method of computing  $\tau_0$  is discussed (the method of geostrophic departures). The data used and sources of error are discussed in sections 3 and 4, respectively. Section 5 is concerned with relationships between  $\tau_0$  and  $V_0$  (surface wind) and  $V_g$  (geostrophic wind). For the data sample of this study,  $V_g$  is a better predictor than  $V_0$  for  $\tau_0$ , and a linear dependence of  $\tau_0$  on wind speed is preferable to a quadratic one. Dependence on Richardson number and temperature advection is examined in section 6. For a given surface or geostrophic wind speed, the stress is larger for warm air advection or high Richardson number than for cold air advection or low Richardson number. The basic data used in this study are presented in the appendix.

2. METHOD OF COMPUTING  $\tau_0$ 

The method is that of "geostrophic departures" used by Sutcliffe (1936), Sheppard and Omar (1952), and others. A short description is relevant here. Ignoring lateral diffusion, one may write the equation of motion as

$$\frac{dV}{dt} = -fk \times (V - V_g) + \frac{1}{\rho} \frac{\partial \tau}{\partial z} \quad (1)$$

where  $V$  is the horizontal wind and  $\tau$  is the horizontal

stress vector. Integrate (1) from the earth's surface ( $z=0$ ) to some height  $z=h$ :

$$\tau_0 = - \int_0^h \rho [fk \times (V - V_g) + \frac{dV}{dt}] dz + \tau_h. \quad (2)$$

In (2),  $\tau_h$  is the stress at  $z=h$ . The height  $h$  will now be identified with the top of the Ekman layer, and  $\tau_h$  will be assumed zero. Furthermore, the acceleration term will be assumed negligible compared to the others. Then equation (2) reduces to

$$\tau_0 = - \int_0^h \rho f k \times (V - V_g) dz. \quad (3)$$

Equation (3) is evaluated by the trapezoidal rule using winds at 500-ft intervals. The level  $h$  is taken as 3,500 ft above terrain. The geostrophic wind is assumed to vary linearly with height as

$$V_g(z) = V_{gSL} + \frac{z+H}{h+H} (V_h - V_{gSL}). \quad (4)$$

Here,  $V_h$  is the wind at  $z=h$ ,  $V_{gSL}$  is the geostrophic wind as measured from the sea-level map, and  $H$  is the height of ground above sea level (that is,  $z=-H$  is sea level).

It would have been possible to obtain the 850-mb geostrophic wind by interpolating in time between maps 12 hr apart. This could then be used for the vertical variation of  $V_g$ . However, this gives rise to considerable error due to the large space and time separation of radiosonde observations. Consequently, it was thought equation (4) would be more accurate. Moreover, (4) gives  $V_g(h) = V_h$ , which is consistent with equation (3).

## 3. DATA USED

Winds were computed by the double theodolite method at five locations near Shilo, Manitoba, during the period Oct. 24 to Nov. 29, 1963 (Danard 1965). The maximum distance between any two of the five stations was 30 mi. The average terrain elevation  $H=1,300$  ft. Observations

were taken on 23 different days, mainly during the daylight hours. During each day, balloons were released at 20-min intervals for about 3 hr at all five locations. At each level, these winds were then vectorially averaged over the five stations and with respect to time. This gave 23 sets of smoothed winds as a function of height. These winds were used to compute  $\tau_0$  by the procedure described in section 2. Radiosonde observations were also taken each day during the 3-hr periods. Synoptic sea-level charts were prepared every 3 hr. In equation (4),  $V_{\rho SL}$  is the average of values obtained from two or three (usually the latter) maps covering each observation period. This tends to reduce errors in  $V_0$  in equation (3). Winds and temperatures at 500-ft intervals, sea-level geostrophic winds, and computed Richardson numbers (see section 6) and surface stresses are given in the appendix.

In addition, for purposes of assessing observational error, simultaneous independent wind measurements were made on Jan. 18, 1964, using two pairs of theodolites.

#### 4. ERRORS IN $\tau_0$

Before discussing the results, it is worthwhile to examine possible errors in  $\tau_0$ . For estimating the magnitude of  $\tau_h$ , omitted in equation (3), it is approximated by

$$\tau_h = A \frac{\Delta V}{\Delta Z} \quad (5)$$

where  $A$  is the momentum exchange coefficient,  $\Delta V$  is the vector wind difference between 3,000 and 4,000 ft above terrain, and  $\Delta Z = 1,000$  ft. When using  $A = 10^2$  gm cm<sup>-1</sup> (Palmén 1959), the average magnitude of  $\tau_h$  during the 23 days is 0.6 dynes cm<sup>-2</sup>.

The effects of wind errors may be examined using the data of Jan. 18, 1964 (see section 3). Since the same  $V_0$  and  $V_0(z)$  are used for both sets of winds, the resulting  $\tau_0$ 's are not strictly independent. However, after using four pairs of simultaneous (not averaged in time) independent wind observations from 500 to 3,500 ft above terrain, the resulting root-mean-square vector error between the four pairs of  $\tau_0$ 's is 0.5 dynes cm<sup>-2</sup>. Since time- and space-averaged winds are used in the main study (Oct. 24 to Nov. 29, 1963), the error resulting from these would likely be less than the January 18 value.

The mean value of  $\tau_0$  during the main observation period is 3.9 dynes cm<sup>-2</sup>. Thus it may be concluded that errors in  $\tau_0$  due to the above sources are usually small compared to  $\tau_0$  itself. However, there may be errors on occasion due to the acceleration term in equation (2) and to uncertainties in  $V_0$ . The method of computing  $V_{\rho SL}$  (see section 3) tends to minimize the latter source of error.

#### 5. DEPENDENCE OF $\tau_0$ ON $V_0$ AND $V_g$

The conventional approach has been to assume that

$$\tau_0 = \rho C V_0^2 \quad (6)$$

TABLE 1.—Coefficients obtained for fitting various equations to  $\tau_0$  (for  $\tau_0$  in dynes cm<sup>-2</sup> and  $V_0, V_g$  in m sec<sup>-1</sup>). The last column is the root-mean-square error in dynes cm<sup>-2</sup> between the observed  $\tau_0$  and the value given by the corresponding regression equation.

Equation	Coefficients	RMS error
$\rho C V_0^2$	$C = 1.3 \times 10^{-1}$	2.1
$\rho C_g V_g^2$	$C_g = 4.7 \times 10^{-3}$	1.8
$a V_0$	$a = 1.4$	1.7
$a_g V_g$	$a_g = 0.39$	1.2
$d V_0^{b+c}$	$d = 1.5, b = 0.5, c = 1.0$	1.7
$d_g V_0^{b_g+c_g}$	$d_g = 0.18, b_g = 1.2, c_g = 0.8$	1.2
$\rho \left( \frac{0.205}{\log_{10} V_g / z_0 - 0.556} \right)^2 V_g^2$	$z_0 = 45$ cm	1.7

or

$$\tau_0 = \rho C_g V_g^2 \quad (7)$$

where  $C$  and  $C_g$  are dimensionless constants (drag coefficients). A relationship such as (7) was proposed by Lettau (1959) (see the last line of table 1). According to summaries by Cressman (1960) and Sawyer (1959), values of  $C$  over land similar to that of southern Manitoba range from  $6 \times 10^{-3}$  (Sutcliffe 1936) to  $5 \times 10^{-2}$  (Buajitti and Blackadar 1957). To Baumgartner (1956), Halstead et al. (1957), Lettau (1950, 1957), and Seeliger (1938) are attributed figures in the range  $1-3 \times 10^{-2}$ . Cressman (1960) also gives a hemispheric map of  $C_g$ . His value for southern Manitoba is  $C_g = 1.5 \times 10^{-3}$ . However, the average values for 23 days of this study are  $C = 1.3 \times 10^{-1}$  and  $C_g = 4.7 \times 10^{-3}$  with  $\rho = 1.15 \times 10^{-3}$  gm cm<sup>-3</sup>. The median values are  $C = 2.0 \times 10^{-2}$  and  $C_g = 2.8 \times 10^{-3}$ . These drag coefficients are thus somewhat larger than the estimates obtained by others. The mean values of  $V_0$  and  $V_g$  during the period are 4.1 and 10.6 m sec<sup>-1</sup>, respectively.

While a quadratic dependence of  $\tau_0$  on wind speed is customarily assumed, results of Mintz (1958) (cited by Cressman 1960) suggest a linear dependence. Figure 1 shows a plot of  $\tau_0$  versus  $V_g$  on a log-log scale. Although there is considerable scatter, it is evident that a linear relation fits these data better than a quadratic one (that is, the slope is closer to 1 than it is to 2). Table 1 shows the results of fitting different curves to the data. In the first four lines of the table, the coefficients are average values for the 23 days. In the last three lines, the coefficients are obtained by regression. In the last line, an equation proposed by Kung (1966) using Lettau's (1962) results is examined. In this equation, the dimensionless ratio  $V_0/z_0 f$  is referred to as the surface Rossby number (Lettau 1959). Two points are immediately obvious. First, equations using  $V_g$  fit the data better than do similar equations using  $V_0$ . Second, a linear dependence gives a better fit than does a quadratic one (fig. 1). Some slight further improvement may be obtained by using the fifth and sixth relations of table 1. However, this is not detectable in the second significant figure of the RMS error. The mean value and standard deviation of  $\tau_0$  are 3.9 and 2.0 dynes cm<sup>-2</sup>, respectively.

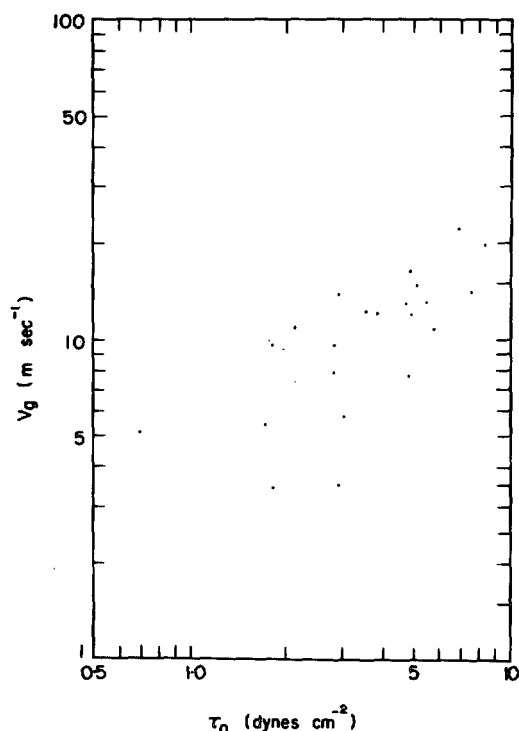


FIGURE 1.—Magnitude of the surface stress ( $\tau_0$ ) versus geostrophic wind ( $V_g$ ), each plotted on a logarithmic scale.

The vector averages of  $\tau_0$ ,  $V_0$ , and  $V_g$  during the 23 days are given in table 2. Note that  $\bar{\tau}_0$  is  $31^\circ$  to the left (counter-clockwise) of  $\bar{V}_0$  and  $40^\circ$  to the left of  $\bar{V}_g$ . The median angles are  $16^\circ$  and  $45^\circ$ , respectively. However, there is considerable variation between individual days, especially when one of the vectors is small. Thus the customary assumption that  $\tau_0$  is parallel to  $V_0$  appears to be only approximately true.

## 6. DEPENDENCE ON RICHARDSON NUMBER AND TEMPERATURE ADVECTION

For each day, the Richardson number

$$Ri = \frac{g(\gamma_a - \gamma)}{T(\partial V / \partial z)^2} \quad (8)$$

is computed. In (8),  $g$  is the acceleration of gravity,  $\gamma_a = g/C_p$  is the dry adiabatic rate of cooling,  $\gamma = -\partial T / \partial z$  is the lapse rate, and  $T$  is the absolute temperature. Lapse rates and shears are calculated over the lowest 100 mb and 3,500 ft, respectively.

The 23 days are divided into two samples according to whether  $Ri$  is large or small. For each sample, the average values of  $a$ ,  $a_s$ ,  $C$ , and  $C_s$  (see table 1) are computed. Results are shown in table 3. The coefficients increase as  $Ri$  increases. However, the small sample size tends to preclude statistical tests of significance. Nevertheless, the coefficients  $a$  and  $C$  both differ significantly at the 5 percent level between the two samples.

For comparison, the 23 days are also separated on the basis of cold or warm air advection (table 4). This is done

TABLE 2.—Average vector  $\tau_0$ ,  $V_0$ , and  $V_g$  during the period of observation

	$\bar{\tau}_0$	$\bar{V}_0$	$\bar{V}_g$
Magnitude	1.7 dynes $\text{cm}^{-2}$	2.5 m $\text{sec}^{-1}$	5.1 m $\text{sec}^{-1}$
Direction	$243^\circ$	$274^\circ$	$292^\circ$

TABLE 3.—Average values of  $a$ ,  $a_s$ ,  $C$ , and  $C_s$  (see table 1) for low and high Richardson numbers

Richardson no.	Low	High
Median Ri	1.4	7.0
Sample size	11	12
$a$	0.77	2.2
$a_s$	0.35	0.42
$C$	$1.3 \times 10^{-2}$	$2.2 \times 10^{-1}$
$C_s$	$2.9 \times 10^{-3}$	$6.2 \times 10^{-3}$

TABLE 4.—Average values of  $a$ ,  $a_s$ ,  $C$ , and  $C_s$  (see table 1) for cold and warm air advection

Advection	Cold	Warm
Sample size	13	10
$a$	1.0	2.2
$a_s$	0.33	0.45
$C$	$4.6 \times 10^{-2}$	$2.1 \times 10^{-1}$
$C_s$	$2.7 \times 10^{-3}$	$7.1 \times 10^{-3}$

TABLE 5.—Observation periods and Richardson numbers  $Ri$  (see section 6)

Day	Date	Time	Ri
1	24 Oct. 1963	1600–1900 GMT	−0.14
2	25	1700–1900	−0.56
3	28	1200–1500	4.0
4	29–30	2300–0200	12.3
5	30	1800–2100	2.3
6	31	1800–2100	3.9
7	1 Nov.	1900–2200	−2.7
8	2	1800–2100	2.7
9	5	1800–1800	13.2
10	6	1600–1800	6.2
11	7–8	2100–0000	7.8
12	8	1700–2000	15.3
13	12	1900–2300	−0.56
14	15	2000–2300	4.5
15	18	1900–2200	2.7
16	19	1900–2200	1.4
17	22	1600–1900	4.6
18	23–24	2200–0100	708
19	24	1600–1800	5.6
20	25	1600–1800	101
21	26	1900–2200	2.8
22	28	1900–2300	2.2
23	29	1800–2100	0.92

simply by noting the directions of  $V_{gSL}$  and  $V$  at 3,500 ft. The latter is assumed equal to the geostrophic value there. The coefficients are larger for warm air advection than for cold.

When comparing tables 3 and 4, it appears that the Richardson number and temperature advection are about equally important in determining the coefficients. The latter may, perhaps, be preferred because of its simplicity.

## 7. CONCLUDING REMARKS

From table 1, it would appear that the best relationship is

$$\tau_0 = a_p V_0 \quad (9)$$

with  $a_p$  perhaps increasing with increasing Richardson number or warm air advection (see table 3). However, since

most workers prefer a quadratic dependence, one is reluctant to make a general claim for the validity of equation (9). Nevertheless, for this one geographical area and the data sample used, the results clearly favor its use.

## APPENDIX—WIND AND TEMPERATURE DATA

The observation intervals and computed Richardson numbers (see section 6) are listed in table 5. The 23 days are then separated into four samples on the basis of cold and warm air advection (as described in section 6) and low and high geostrophic wind speeds. The median value of  $V_{SL}$  is 11.0 m sec<sup>-1</sup>. Winds, temperatures, and computed stresses are presented in tables 6–9.

TABLE 6.—The  $\tau_0$  (dynes cm<sup>-2</sup>),  $V_{SL}$  (m sec<sup>-1</sup>),  $V(h)$  (m sec<sup>-1</sup>), and temperature  $T$  (deg C) for cases with cold air advection and  $V_{SL} \leq 11.0$  m sec<sup>-1</sup>.  $\beta$  and  $M$  denote vector direction (deg) and magnitude, respectively. See table 5 for key to day number. The  $h$  is height (ft) above the earth's surface.

Day		$\tau_0$	$V_{SL}$	$V(h)$										
				0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
4	$\beta$	306.6	316.0	270.0	265.4	279.0	294.5	306.5	314.9	309.1	306.2	308.4	312.4	312.3
	$M$	1.7	5.2	0.7	5.7	5.9	6.2	6.5	6.2	5.8	5.4	5.9	6.8	7.0
	$T$			6.1	8.2	7.6	6.8	6.0	5.3	4.4	3.5	2.4	1.4	0.1
5	$\beta$	253.7	307.0	270.0	261.2	264.3	273.0	280.1	280.9	278.4	275.7	275.5	275.1	274.5
	$M$	2.8	9.4	6.0	7.1	7.5	9.1	10.8	11.2	11.0	11.3	10.3	10.3	10.4
	$T$			10.3	9.3	8.0	6.5	5.2	5.0	4.1	2.9	1.6	0.2	-1.3
10	$\beta$	256.0	333.0	280.0	305.9	312.5	307.1	304.3	299.3	295.9	299.1	300.3	299.7	296.4
	$M$	1.0	4.8	2.0	4.4	5.9	6.8	7.2	7.6	8.0	8.7	9.1	9.1	9.0
	$T$			2.1	-0.3	0.6	1.6	2.5	2.1	1.6	1.1	0.2	-0.6	1.5
17	$\beta$	318.2	2.6	315.0	314.1	325.5	335.3	331.8	323.9	316.7	317.6	316.8	314.3	310.0
	$M$	2.1	11.0	5.3	9.2	11.0	12.4	12.3	12.4	13.0	13.1	12.5	14.2	15.9
	$T$			-12.1	-13.4	-14.2	-14.1	-13.2	-14.0	-14.0	-13.0	-13.4	-13.4	-12.4
18	$\beta$	109.5	142.7	45.0	73.8	96.6	116.0	120.7	105.3	95.9	12.1	340.5	329.9	322.0
	$M$	2.8	10.8	1.4	4.6	4.4	5.3	4.3	3.2	1.6	1.0	3.2	5.7	6.6
	$T$			-14.3	-15.2	-15.9	-16.4	-15.6	-14.7	-12.2	-9.2	-8.8	-8.9	-9.3
23	$\beta$	256.1	348.0	315.0	310.6	321.6	328.2	331.8	332.0	330.5	328.7	327.7	329.6	333.1
	$M$	5.7	9.1	4.1	5.9	5.8	6.6	8.9	12.1	14.4	15.9	16.6	18.7	19.0
	$T$			-1.0	-1.8	-2.7	-3.5	-4.4	-5.2	-6.1	-7.1	-8.0	-9.0	-10.1

TABLE 7.—Same as table 6 but for cold air advection and  $V_{SL} > 11.0$  m sec<sup>-1</sup>

Day		$\tau_0$	$V_{aSL}$	$V(h)$										
				0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
1	$\beta$	234.2	301.4	247.5	261.5	261.7	263.2	266.9	270.9	271.6	271.6	270.3	269.6	269.9
	$M$	8.2	19.3	8.4	14.2	14.3	15.1	16.3	18.7	20.5	21.8	22.0	23.0	23.7
	$T$			11.8	10.3	8.9	7.4	5.9	4.3	2.8	2.5	2.8	2.3	1.0
2	$\beta$	222.4	277.5	247.5	251.8	252.7	253.6	254.6	256.0	257.3	260.2	261.1	262.3	262.9
	$M$	6.9	22.9	9.3	16.8	17.8	18.1	18.0	18.4	18.9	19.9	19.1	18.8	21.6
	$T$			16.8	15.2	13.7	12.0	10.2	8.6	6.9	5.8	4.6	3.4	2.2
3	$\beta$	240.9	338.0	270.8	324.5	343.6	348.3	340.1	333.7	332.4	331.1	330.7	330.4	330.3
	$M$	4.7	12.2	4.5	8.2	9.4	8.2	9.1	11.9	13.5	14.0	14.4	14.0	13.7
	$T$			-4.8	-1.3	-0.6	-1.2	-2.0	-2.0	-1.2	-2.4	-3.6	-5.0	-6.3
6	$\beta$	283.4	334.0	306.5	303.7	306.0	307.6	309.3	312.1	313.8	315.2	316.7	318.4	321.9
	$M$	3.8	12.9	6.4	9.1	9.2	9.4	9.5	9.6	9.6	9.8	10.1	10.4	11.0
	$T$			7.8	6.2	4.9	3.7	2.7	1.5	0.4	-1.3	-3.6	-5.7	-7.7
13	$\beta$	312.5	17.7	327.5	351.9	350.9	348.1	345.6	348.6	355.1	4.5	8.4	9.3	9.0
	$M$	7.5	13.4	4.8	9.2	9.7	9.9	10.3	11.1	12.3	15.4	17.6	16.9	16.5
	$T$			1.9	-2.0	-3.2	-4.6	-6.1	-7.3	-8.8	-10.2	-9.6	-8.8	-9.5
15	$\beta$	302.0	339.0	287.0	285.5	283.6	281.6	276.8	270.8	267.7	266.7	266.1	265.5	264.2
	$M$	3.5	13.1	7.5	10.9	11.6	13.6	18.1	19.5	18.6	18.5	17.9	18.4	19.0
	$T$			2.3	1.0	-0.2	-1.7	-1.4	2.0	1.6	0.4	-0.5	-1.6	-2.6
21	$\beta$	308.5	336.0	286.0	298.8	308.6	315.0	322.0	322.2	322.5	317.8	319.5	321.2	319.7
	$M$	2.9	12.6	5.5	12.8	15.5	17.3	17.2	17.2	16.5	17.6	18.3	20.7	21.9
	$T$			6.2	5.5	4.8	4.3	3.6	4.7	7.6	9.2	7.9	6.6	5.3

TABLE 8.—Same as table 6, but for warm air advection and  $V_{eSL} \leq 11.0 \text{ m sec}^{-1}$ 

Day	$\tau_0$	$V_{eSL}$	$V(h)$										
			0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
7	$\beta$	252.0	247.0	269.7	283.9	284.0	282.6	285.7	289.2	298.1	309.3	319.1	326.6
	$M$	0.7	2.4	1.8	2.9	3.0	3.1	3.1	3.1	3.7	4.7	6.0	7.2
	$T$			7.2	5.4	3.6	1.8	0.0	-1.7	-2.7	-3.7	-4.6	-5.6
9	$\beta$	192.3	114.0	101.0	106.4	116.9	129.9	158.3	202.1	225.3	248.1	255.8	268.8
	$M$	2.9	5.6	1.5	5.3	5.8	4.8	3.2	2.9	3.1	4.2	6.0	7.3
	$T$			3.2	2.9	7.8	7.0	6.9	6.3	5.6	4.5	3.4	2.6
11	$\beta$	239.3	242.0	308.0	233.5	228.9	228.0	235.8	234.6	268.4	270.0	267.3	264.0
	$M$	3.0	5.0	1.3	3.9	5.3	6.5	8.2	8.6	8.7	8.0	7.5	7.2
	$T$			6.3	7.6	6.6	5.6	6.3	7.2	6.6	5.6	4.3	3.5
12	$\beta$	17.5	132.0	67.0	174.0	212.4	211.1	212.8	214.6	216.1	216.8	218.6	231.2
	$M$	1.8	4.2	0.4	0.8	2.5	4.0	4.3	4.5	4.4	4.1	4.2	3.7
	$T$			4.2	3.5	5.4	5.4	4.6	3.7	2.6	1.6	0.6	-0.4
14	$\beta$	100.7	149.0	75.0	78.9	124.1	211.3	223.4	227.4	227.2	229.7	232.4	233.4
	$M$	4.8	9.2	1.6	3.5	2.4	4.4	6.9	7.3	8.4	9.7	10.5	10.6
	$T$			3.7	3.8	4.0	4.9	5.1	5.3	7.4	8.2	9.0	9.2
16	$\beta$	58.4	143.0	145.4	166.1	177.2	204.8	221.0	221.6	221.0	220.6	220.6	221.4
	$M$	1.8	9.6	5.2	6.5	7.5	9.8	15.3	19.7	19.5	18.1	18.0	17.2
	$T$			4.3	3.0	1.7	0.9	3.4	4.7	5.6	4.6	3.9	2.9

TABLE 9.—Same as table 6 but for warm air advection and  $V_{eSL} > 11.0 \text{ m sec}^{-1}$ 

Day	$\tau_0$	$V_{eSL}$	$V(h)$										
			0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
8	$\beta$	175.9	215.5	179.2	190.2	195.8	207.3	222.1	231.2	234.1	238.6	243.4	249.0
	$M$	5.1	15.2	5.9	9.0	9.7	10.4	12.2	14.7	15.4	13.6	12.8	12.8
	$T$			10.4	9.2	7.7	6.3	5.1	8.0	9.8	9.8	8.4	7.3
19	$\beta$	155.4	184.0	135.0	148.9	167.1	177.1	186.7	191.8	200.6	203.7	206.9	208.5
	$M$	4.8	17.8	3.5	8.2	14.0	17.4	15.6	13.3	11.5	12.2	11.2	11.2
	$T$			-9.3	-9.8	-9.8	-8.5	-5.3	-3.0	-2.5	-2.8	-3.5	-4.2
20	$\beta$	163.6	224.0	235.0	239.0	223.9	192.6	199.4	217.8	245.5	272.6	275.2	283.2
	$M$	4.9	15.2	0.8	6.1	6.3	4.6	3.7	3.4	2.9	3.8	5.6	7.5
	$T$			-15.8	-14.4	-12.7	-8.1	-5.0	-5.2	-5.4	-4.3	-2.9	-4.0
22	$\beta$	324.3	329.2	292.0	298.7	301.6	307.8	314.1	319.2	324.5	329.8	333.6	335.6
	$M$	5.4	12.5	7.6	12.0	14.0	15.1	15.8	15.2	14.5	14.0	13.0	12.4
	$T$			-1.4	-2.6	-3.8	-5.1	-6.3	-6.8	-6.6	-7.6	-8.4	-9.4

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#### CORRECTION NOTICE

Vol. 98, No. 4, Apr. 1970: p. 315, eq. (1),  $y$  is to be read instead of  $Y$ ; p. 316, eq. (3),  $-f\bar{u}$  instead of  $-fu$ ; eq. (5) first line,  $\bar{v}^2$  instead of  $\bar{v}^2$ , and  $\bar{v}\bar{\omega}$  instead of  $\bar{v}\bar{\omega}$ ; eq. (7), left col., first line,  $\bar{v}^*$  instead of  $\bar{u}^*$ ; eq. (7), right col., first line,  $\bar{v}^*$  instead of  $\bar{u}^*$ ; eq. (8),  $V$  instead of  $v$  in two places.